Cognitive Goals

Every mathematical sciences major should be designed to help students acquire "mathematical habits of mind." Students should develop the ability and inclination to use precise language, reason carefully, solve problems effectively, and use mathematics to advance arguments and increase understanding. These cognitive goals are not achieved in a single assignment or course; they must be approached within the context of each student's mathematical maturation throughout his or her undergraduate years. A well-constructed curriculum supports students in learning concepts, acquiring skills, and achieving cognitive goals. In the following paragraphs we describe several cognitive goals in more detail.

Cognitive Recommendation 1: *Students should develop effective thinking and communication skills.*

Major programs should include activities designed to promote students' progress in learning to

- state problems carefully, articulate assumptions, understand the importance of precise definition, and reason logically to conclusions;
- identify and model essential features of a complex situation, modify models as necessary for tractability, and draw useful conclusions;
- deduce general principles from particular instances;
- use and compare analytical, visual, and numerical perspectives in exploring mathematics;
- assess the correctness of solutions, create and explore examples, carry out mathematical experiments, and devise and test conjectures;
- recognize and make mathematically rigorous arguments;
- read mathematics with understanding;
- communicate mathematical ideas clearly and coherently both verbally and in writing to audiences of varying mathematical sophistication;
- approach mathematical problems with curiosity and creativity and persist in the face of difficulties;
- work creatively and self-sufficiently with mathematics.

Cognitive Recommendation 2: *Students should learn to link applications and theory.*

Mathematics students should encounter a range of contemporary applications that motivate and illustrate the ideas they are studying, become aware of connections to other areas (both in and out of the mathematical sciences), and learn to apply mathematical ideas to problems in those areas. Students should come to see mathematical theory as useful and enlightening in both pure and applied contexts.

Cognitive Recommendation 3: *Students should learn to use technological tools.*

Mathematical sciences major programs should teach students to use technology effectively, both as a tool for solving problems and as an aid to exploring mathematical ideas. Use of technology should occur with increasing sophistication throughout a major curriculum.

Cognitive Recommendation 4: *Students should develop mathematical independence and experience open-ended inquiry.*

A mathematical sciences major should be structured to move students beyond the carefully choreographed mathematical experiences of the classroom. A major curriculum should gradually prepare students to pursue open-ended questions and to speak and write about mathematics with increasing depth and sophistication.

Content Goals

The possible variety of major programs makes "core" content difficult to specify. Nevertheless, we do recommend that all programs in the mathematical sciences include several common elements. These recommendations were informed by broad consultation, including reports from study groups considering courses and programs, and discussions with directors of both Ph.D. and professional master's programs. These content goals connote mathematical subjects and ideas that can appear in a curriculum in various ways---not necessarily in specific courses with the same names.

Content Recommendation 1: *Mathematical sciences major programs should include concepts and methods from calculus and linear algebra.*

Every major in the mathematical sciences draws on ideas and methods from calculus and linear algebra. Calculus is the foundation of the study of continuous processes, and thus a gateway to the physical, biological, and social sciences. Instruction in calculus should include consideration of both single- and multivariable approaches. Linear algebra, the study of multivariate linear systems and transformations, is essential preparation for advanced work in the sciences, statistics, and computing. Linear algebra also introduces students to discrete mathematics, algorithmic thinking, a modicum of abstraction, moderate sophistication in notation, and simple proofs. Combining ideas from linear algebra and calculus helps students develop facility with visualization, see connections among mathematical areas, and appreciate the power of abstract thinking.

Content Recommendation 2: *Students majoring in the mathematical sciences should learn to read, understand, analyze, and produce proofs at increasing depth as they progress through a major.*

Proofs are indispensable to the practice and culture of mathematics. Students in all mathematics courses, whether or not for majors, should encounter elements of mathematical argument, precision, and justification. All mathematical science majors should learn to read, understand, analyze, and produce proofs, at increasing depth as they progress through a major. Individual departments may foster this development as institutionally appropriate: through dedicated "transition" courses or by distributing this content among several courses.

Content Recommendation 3: *Mathematical sciences major programs should include concepts and methods from data analysis, computing, and mathematical modeling.*

Working mathematicians often face quantitative problems to which analytic methods do not apply. Solutions often require data analysis, complex mathematical models, simulation, and tools from computational science. To meet these workplace expectations every mathematical sciences major should have, at a minimum:

- a command of data analysis and statistical inference at a level equivalent to that attained in an applied data analysis course;
- experience working with professional-level technological tools such as computer algebra systems, visualization software, and statistical packages;
- modest experience writing computer programs;
- experience tackling ill-posed real-world problems by building and analyzing appropriate deterministic and stochastic mathematical models.

Content Recommendation 4: *Mathematical sciences major programs should present key ideas and concepts from a variety of perspectives to demonstrate the breadth of mathematics.*

Programs should present key ideas from a variety of perspectives, employ a broad range of examples and applications to motivate and illustrate the material, promote awareness of connections to subjects both within and beyond the mathematical sciences, and strengthen each student's ability to apply the course material to these subjects. Programs should introduce historical and contemporary topics and applications, highlighting the vitality and importance of modern mathematics, and the contributions of diverse cultures.

Content Recommendation 5: *Students majoring in the mathematical sciences should experience mathematics from the perspective of another discipline.*

Applications of mathematics to other fields continue to evolve and expand. Mathematics students should encounter substantive applications throughout the curriculum. When possible, these applications should include perspectives of non-mathematicians who use mathematics to clarify or extend their own subject.

Content Recommendation 6: *Mathematical sciences major programs should present key ideas from complementary points of view: continuous and discrete; algebraic and geometric; deterministic and stochastic; exact and approximate.*

Students acquire mathematical depth and perspective by encountering and employing a variety of mathematical viewpoints:

- **Continuous and discrete:** Continuous mathematics—calculus, analysis, and differential equations—has long been central to mathematics major curricula. In recent decades, however, advances in computer science, operations research, mathematical modeling, and data analysis have dramatically increased the importance of discrete mathematics. Techniques from discrete mathematics—difference equations, recursive methods, combinatorial arguments, graph-theoretic models—prepare students to encounter the rich interplay between continuous and discrete approaches in future study and careers. Discrete mathematics should therefore take its place alongside continuous mathematics as a crucial element of undergraduate study.

- **Algebraic and geometric:** Algebra and algebraic structures—vector spaces, groups, rings, fields—are fundamental to mathematics and should be included in every undergraduate mathematics curriculum. Geometry and visualization are different ways of thinking and provide an equally important perspective. A geometry course is a useful part of any student's major program and is essential for future high school teachers. But geometric viewpoints should appear beyond geometry courses. Geometric reasoning and visualization complement algebraic thinking in linear algebra and multivariable calculus, and remain important in more advanced courses such as real analysis and differential equations.

- **Deterministic and stochastic:** New applications of mathematics to the biological and social sciences make understanding and modeling randomness and random phenomena increasingly important in undergraduate mathematics. Students should see the need for and master basic methods associated with elementary discrete- and continuous-time stochastic models. Deterministic models can fruitfully represent large-scale behavior, but random variation is often better seen through the stochastic lens. Stochastic applications appear naturally in statistics courses, but can also be introduced in courses such as elementary probability, matrix algebra, or graph theory. They can be explored further in courses such as differential equations or mathematical models.

- **Exact and approximate:** Students should be able to find, use, and evaluate approximations, understanding that exact answers, although desirable, are often unavailable or impractical. Students should be introduced to numerical methods, discrete approximations of continuous phenomena (and vice versa), and the general role of approximation in solving problems.

Balancing a variety of mathematical perspectives need not mean devoting separate courses to each. A more feasible strategy is to structure the curriculum and requirements intentionally to include these themes over the course of a student's major. For instance, modern computing tools—which themselves combine graphical, numerical, and algebraic resources—can facilitate presenting these viewpoints in many undergraduate courses.

Content Recommendation 7: *Mathematical sciences major programs should require the study of at least one mathematical area in depth, with a sequence of upper-level courses.*

Mathematics grows through the construction of abstract theories from definitions, examples, and theorems. Students learn to cope with such complexity by grappling with clusters of related ideas, in depth and over an extended period. Every mathematics major student should encounter at least one area in depth, drawing on ideas and tools from previous courses and making connections among them. Departments can meet this goal by requiring either two related courses or a year-long sequence at the upper level. This goal prescribes neither a particular area of study nor whether the material be mainly theoretical or abstract; possibilities include Probability and Mathematical Statistics, Real Analysis I/II, and Abstract Algebra I/II.

Content Recommendation 8: *Students majoring in the mathematical sciences should work, independently or in a small group, on a substantial mathematical project that involves techniques and concepts beyond the typical content of a single course*.

Every major student should have a "high impact" experience that requires substantial work in mathematics outside the carefully scripted confines of ordinary course work. Students should present their results in written and oral forms. Institutions can provide this opportunity in various ways: undergraduate research experiences, courses driven by inquiry or open-ended problem solving, capstone courses, internships or jobs with a substantial mathematical component, etc.

Content Recommendation 9: *Mathematical sciences major programs should offer their students an orientation to careers in mathematics.*

Helping students explore professional options can improve recruitment and retention of majors. Because degree programs vary, departments should tailor these messages in locally appropriate ways. Departments can employ both "static" information—posters, links on department websites, career brochures, career information from professional societies—and more dynamic strategies: inviting external speakers (including alumni); short, self-contained, career-focused orientations or assignments within courses; and encouraging and supporting students' attendance at regional and national professional meetings.

Review and Renewal

A healthy mathematical sciences program should incorporate intentional evolution and continual improvement. Every mathematical sciences department should have and follow a strategic plan that acknowledges local conditions and resources, but is also informed by recommendations from the greater mathematical community. The process of planning and renewal should be guided by consultation both within the department and with outside stakeholders at the institution. Departments should assess their progress in meeting cognitive and content goals through systematic collection and evaluation of evidence. Two documents from the MAA Committee on Department Review, *Guidelines for Undertaking a Self Study in the Mathematical Sciences* and *Guidelines for Serving as a Consultant in the Mathematical Sciences,* provide valuable guidance to a department engaged in review and renewal.

A suggested framework for continuous improvement

Initially, departments should select a mix of long-term (five years, say) goals and short-term (one or two years) objectives and choose specific strategies for meeting long- and short-term objectives. Then, departments should regularly

1. collect data on success in meeting departmental goals (e.g., placement, persistence, success in downstream courses, ability to communicate mathematics);
2. assess whether departmental teaching and pedagogy effectively support departmental goals;